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Fractional Calculus and Special Functions with Applications in Applied Mathematics and Other Sciences-A review

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Abstract: -

Fractional differential equations and other issues involving certain mathematical physics functions, as well as their expansions and generalisations in one or more variables, are typically the outcome of mathematical modelling of real-life difficulties. Additionally, fractional order PDEs are responsible for controlling the majority of physical events in many other models, including those of fluid dynamics, quantum physics, electricity, ecological systems, and many more. It is crucial to be conversant with all existing and newly developed techniques for solving fractional order PDEs, as well as the applications of these techniques. On the basis of various definitions of various fractional derivatives and various fractional integrals, explicit equations and graphs of a few special functions are obtained in this paper. Their applications are also reviewed in the paper.

1. Introduction:

Fractional Calculus is a new powerful tool which has been recently employed to model complex biological systems with non-linear behavior and long-term memory. In spite of its complicated mathematical background, fractional calculus came into being of some simple questions which were related to the derivation concept; such questions as while the first order derivative represents the slope of a function, what a half order derivative of a function reveals about it? Finding answers to such questions, scientists managed to open a new window of opportunity to mathematical and real world, which has arisen many new questions and intriguing results [1]. For example, the fractional order derivative of a constant function, unlike the ordinary derivative, is not always zero. In this tutorial-based paper it is sought to answer the aforementioned questions and to construct a comprehensive picture of what fractional calculus is, and how it can be utilized

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for modelisation purpose. After an extensive literature review of the concepts and application of this potent tool, a novel application of this tool is developed for simulating a dynamic system in order to investigate the mechanical behavior of a cell [2,3].

Riemann had arrived at expression for fractional integration as [4]

$\frac{1}{\Gamma(2)}\int_0^u \frac{f(t)dt}{(u-t)^{1-a}}$, **u > 0**

This paper was published in 1876.

Most of the theory of fractional calculus is based upon the familiar differential operator defined as, [5]

$$D^w_{x\{f(x)\}} = \begin{cases} \frac{1}{\Gamma w}\int_r^x (x-t)^{w-1} f(t)dt & , Re(w) > 0 \\ \frac{d^r}{dx^r}\left[{}_r\ D^w_{x\{f(x)\}}\right], & 0 \le Re(w) < r \end{cases} , \qquad \dots (1.1)$$

where r is a positive integer.

Case(i): If $r = o$,the (i) reduces to classical Riemann-Liouville fractional derivatives or integral of order w.

Case(ii): If $r \to \infty$,the equation (i) may be defined with the definition of the familiar Wayl fractional operator of order w.

Mishra (1981) has defined the fractional derivative operator in the following manner

$$D^\alpha_x(x^{\mu-1}) = \frac{d^\alpha}{dx^\alpha} x^{\mu-1} = \frac{\Gamma(\mu)}{\Gamma(\mu-\alpha)} x^{\mu-\alpha-1}, \quad \alpha \neq \mu \qquad \dots (1.2)$$

$$D_{k,\alpha,x}(x^\mu) = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} x^{\mu+k}, \quad \alpha \neq \mu+1 \qquad \dots (1.3)$$

Erdélyi–Kober fractional integral operator

$$I^+\{f(x)\} = \frac{t^{-\mu-\lambda}}{\Gamma\mu}\int_0^t x^\gamma (t-x)^{\mu-1} f(x)dx\ [Re(\mu) > 0, \gamma > -1]$$

$$K^-\{f(x)\} = \frac{t^\delta}{\Gamma\mu}\int_t^\infty t^{-\delta-\mu}(x-t)^{\mu-1} f(x)dx\ [Re(\mu) > 0, \delta > -1]$$

These operators are generalization of Riemann-Liouville and Weyl fractional integral operator.

2. Review of Literature:

Motivated essentially by the success of the applications of the [6] Mittag-Leffler functions in many areas of science and engineering, the authors present in a unified manner, a detailed account or rather a brief survey of the Mittag-Leffler function, generalized Mittag-Leffler functions, Mittag-Leffler type functions, and their interesting and useful properties. Applications of Mittag-Leffler functions in certain areas of physical and applied sciences are

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also demonstrated. In this survey paper, nearly all types of Mittag-Leffler type functions existing in the literature are presented. An attempt is made to present nearly an exhaustive list of references concerning the Mittag-Leffler functions to make the reader familiar with the present trend of research in Mittag-Leffler type functions and their applications. [7]

Here one shows that if one considers fractional derivative via fractional difference, then this equation might have solutions which are nowhere differentiable. Various formulae are derived with respect to this question, and various open problems are outlined. [8]

Derive the solution of a fractional differential equation associated with a RLC electrical circuit with order $1 < a \leq 2$ and $1 < b \leq 1$. The Sumudu transform technique is used to derive the solution. The results are derived here new and compact forms in terms of the generalized Mittag-Leffler function, which are suitable for numerical computation. [9]

Now the bounds of fractional integral operators containing an extended generalized Mittag-Leffler function as a kernel via several kinds of convexity. In particular, the established bounds are studied for convex functions and further connected with known results. Furthermore, these results applied to the parabolic function and consequently recurrence relations for Mittag-Leffler functions are obtained. Moreover, some fractional differential equations containing Mittag-Leffler functions are constructed and their solutions are provided by Laplace transform technique. [10]

After that fractional calculus dates its inception to a correspondence between Leibniz and L'Hospital in 1695, when Leibniz described "paradoxes" and predicted that "one day useful consequences will be drawn" from them. In today's world, the study of non-integer orders of differentiation has become a thriving field of research, not only in mathematics but also in other parts of science such as physics, biology, and engineering: many of the "useful consequences" predicted by Leibniz have been discovered. However, the field has grown so far that researchers cannot yet agree on what a "fractional derivative" can be. [11].

Variable-order fractional operators were conceived and mathematically formalized only in recent years. The possibility of formulating evolutionary governing equations has led to the successful application of these operators to the modelling of complex real-world problems ranging from mechanics, to transport processes, to control theory, to biology. Variable-order fractional calculus (VO-FC) is a relatively less known branch of calculus that offers remarkable opportunities to simulate interdisciplinary processes [12].

Recognizing this untapped potential, the scientific community has been intensively exploring applications of VO-FC to the modelling of engineering and physical systems. This review is intended to serve as a starting point for the reader interested in approaching this fascinating field. Here provided a concise and comprehensive summary of the progress made in the development of VO-FC analytical and computational methods with application to the simulation of complex physical systems. [13]

As we know fractional calculus was born in 1695 on September 30 due to a very deep question raised in a letter of L'Hospital to Leibniz. The prophetical answer of Leibniz to that deep question encapsulated a huge inspiration for all generations of scientists and is continuing to stimulate the minds of contemporary researchers. During 325 years of existence, fractional calculus has kept the attention of top-level mathematicians, and during the last period of time it has become a very useful tool for tackling the dynamics of complex systems from various branches of science and engineering. [14]

Now modifying the (k, s) fractional integral operator involving k-Mittag-Leffler function and discuss its properties. We originate a new fractional operator named (k, s)-Prabhakar

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derivative and obtained some classical fractional operators as a special case of the newly proposed derivative. Some properties of the introduced operator are also part of the present work. The generalized Laplace transform is employed to study the characteristics of fractional operators. Here modeled the free-electron laser (FEL) equation by involving the proposed derivative and can find the solution by using the said Laplace transform. [15]

A new fractional derivative with a non-singular kernel involving exponential and trigonometric functions is proposed in this paper. The suggested fractional operator includes as a special case Caputo-Fabrizio fractional derivative. Theoretical and numerical studies of fractional differential equations involving this new concept are presented. Next, some applications to RC-electrical circuits are provided. [16]

The Saigo's k-fractional integral and derivative operators involving k-hypergeometric function in the kernel are applied to the generalized k-Bessel function; results are expressed in term of k-Wright function, which are used to present image formulas of integral transforms including beta transform. Also, special cases related to fractional calculus operators and Bessel functions are considered. [17]

3. The Multivariable H-Function:

The multivariable Weyl fractional integral operator is defined as follows [18]

$$W^{\mu_1,\ldots\ldots,\mu_k}\{g(x_1 \ldots . x_k; u_1 \ldots u_k): v_1 \ldots v_k; z_1 z_2 \ldots z_k\}$$
$$= \int_{v_1}^{\infty} \ldots \ldots \int_{v_k}^{\infty} \prod_{j=1}^{k} \left\{\frac{\left(x_j - v_j\right)^{\mu_{j-1}}}{\Gamma\mu_j}\right\} H^{0,u:m_1 n_1;\ldots;m_k,n_k}_{p,q:p_1 q_1;\ldots;p_k,q_k}\left(\begin{matrix} z_1(x_1-v_1)^{\sigma_1} \\ z_k(x_k-v_k)^{-\sigma_k}\end{matrix}\right) g(x_1 \ldots x_k; u_1 \ldots u_k) dx \ldots dx_k \quad \ldots (2.1)$$

Provided that the integral on right hand on right hand side of (2.1) converges absolutely.

In (2.1) and elsewhere $H[z_1 z_2 \ldots z_k]$ stands for the multivariable H-function introduced by H.M Srivastava and R. Panda through a series of research papers [50]. This function is defined and represented in the following manner

$$H[z_1 \ldots z_k] = H^{0,n:\{m_k,n_k\}}_{p,q\ :\{p_k,q_k\}}\left([z_1 \ldots z_k]\left[\begin{matrix}\left(a_j; \alpha_j' \ldots, \alpha_j^{(k)}\right)_{1,p} : \left\{\left(c_j^{(k)}, \gamma_j^{(k)}\right)_{1,p_k}\right\} \\ \left(b_j; \beta_j' \ldots, \beta_j^{(k)}\right)_{1,q} : \left\{\left(d_j^{(k)}, \delta_j^{(k)}\right)_{1,q_k}\right\}\end{matrix}\right]\right)$$
$$= \frac{1}{(2\pi\omega)^k} \int_{L_1} \ldots \int_{L_r} \psi(\xi_1 \ldots \xi_k) \prod_{i=1}^{k} \left\{\phi_i(\xi_i) z_i^{\xi_i} d\xi_i\right\} \ldots (2.2)$$

where $\omega = \sqrt{-1}$, and

$$\psi(\xi_1 \ldots \xi_k) = \frac{\prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{k} \alpha_j^{(i)} \xi_i)}{\prod_{j=n+1}^{p} \Gamma(a_j - \sum_{j=1}^{k} \alpha_j^{(i)} \xi_i) \prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{k} \beta_j^{(i)} \xi_i} \quad \ldots (2.3)$$

and

$$\phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i} \Gamma(d_{j-}^{(i)} \delta_j^{(i)} \xi_i) \prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i)}{\prod_{j=m+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i) \prod_{j=n+1}^{p_i} \Gamma(c_j^{(i)} - \gamma_j^{(i)} \xi_i)} \forall i \epsilon \{1,2, \ldots . k\} \ldots (2.4)$$

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and an empty product is interpreted as unity.

$n, m_{i,} n_{i,} p_{i,} q_i$ are integers such that $o \leq n \leq$p, $1 \leq m_i \leq q_i$, $q \geq 0$ and $1 \leq n_i \leq p_i$,$\forall i \epsilon \{1,2, \ldots . k\}$. The coefficients $\alpha_j^i, j = 1,2, \ldots, p; \gamma_j^i, j = 1,2, \ldots, p_i$; $\beta_j^i, j = 1,2, \ldots, q$; $\delta_j^i, j = 1,2, \ldots, q_i$ are real number.

The contour L_i in the complex ξ_i in plane is of the Mellin-Barnes type which varies from $-\omega\infty t0 + \omega\infty$ with indentation, if necessary ,in such a manner that all the poles of $\Gamma(d_{j-}^{(i)} \delta_j^{(i)} \xi_i)$, j = 1,...,$m_i$, are to the right and those of $\Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i\right)$,j=1,...,$n_i$ and $\Gamma(1 - a_j - \sum_{i=1}^{k} \alpha_j^{(i)} \xi_i)$, j = 1,…,n to the left ofξ_i,the various parameters being so restricted that these poles are all simple and none of them coincide. And with points z_i = 0, $\forall i \epsilon \{1,2, \ldots . k\}$ are being tacitly excluded, the multiple integral of equation (1) converges if

$$|\arg (z_i)| < \frac{1}{2} \pi T_i ,\forall i \epsilon \{1,2, \ldots . k\} \qquad \ldots (2.6)$$

where $\Delta_i = - \sum_{j=n+1}^{p} \alpha_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n+1}^{p_i} \gamma_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=m+1}^{q_i} \delta_j^{(i)} > 0$
$\forall i \epsilon \{1,2, \ldots . k\}$ ... (2.7)

and also $\nabla_i = \sum_{j=1}^{p} \alpha_j^{(i)} + \sum_{J=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^{q} \beta_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} \leq 0$ ... (2.8)

whenever there is no ambiguity or confusion, we shall use a contracted notation and write first member of equation (2.2) in following abbreviated form

$$H_{p,q;(p_1,q_1);\ldots;(p_k,q_k)}^{0,n;(m_1,n_2);\ldots;(m_k,n_k)} [Z_1,\ldots,Z_r] \qquad \ldots (2.9)$$

or H $[Z_1,\ldots,Z_r]$

further we may recall the known asymptotic expansion in the following form $H_{p,q;(p_1,q_1);\ldots;(p_k,q_k)}^{0,n;(m_1,n_2);\ldots;(m_k,n_k)}$

$$[Z_1,\ldots,Z_r] = \begin{cases} 0(|z_1|^{\tau_1} \ldots, |z_k|^{\tau_k}), \max\{|z_1|, \ldots, |z_k|\} \to 0 \\ 0(|z_1|^{\rho_1}, \ldots, |z_k|^{\rho_k}), n = 0, \min\{\{|z_1|, \ldots, |z_k|\} \to \infty \end{cases}$$

$\ldots (2.10)$

where τ_i = min {Re $(d_j^{(i)}, \delta_j^{(i)})$, j = 1,…,$m_i$ } and ρ_i = max {Re$(c_j^{(i)}$-1), $v_j^{(i)}\}, j = 1, \ldots, n_i\}$

again throughout the present work, we employ in abbreviation (a) to denote the sequence of P parameters a_i,…,a_p: for each i = 1,…,k;$(c^{(i)})$ abbreviated the sequence of P_i parameters c_j^i , j = 1, …,$P^{(i)}$, with similar interpretation for (c), $(c^{(i)})$, etc. , j= 1,…,k; will be understood for example that $c= c^1,c^2,\ldots$ and so on.

Also, for the sake of brevity we use the following contracted notations,

$$[(a)]_l = \prod_{j=1}^{p} [a_j]_l \quad ; \quad \left[(c^{(i)})\right]_l = \sum_{j=1}^{p_i} [c_j^i]_l \quad , i \epsilon \{1,2, \ldots . k\}\ etc \qquad \ldots (2.11)$$

where a_n is the Pocchammer symbol defined by –

$$[\alpha]_l = \frac{\Gamma(\alpha+n)}{\Gamma(a)} = \begin{cases} 1: \ if l=0 \\ a(a+1)\ldots(a+l-1): \ if l=1,2\ldots \end{cases}$$

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4. The I-Function:

Recently development in theory of hypergeometric functions has gained much interest due to introduction of certain new generalized forms of hypergeometric functions. These functions are Mac-Robert's E – function, Meijer's G – function, Fox's H- function and recently I – function [19,20].

The I – function has been defined by Saxena [19]in the course of the solution of dual integral equation involving H – function as kernels and was further studied by Verma, Jain, Vaishya [21] The I – function introduced by Saxena (1982), is defined as -

$$I(Z) = I^{m,n}_{p_i q_{i;r}}\left[Z \left| \begin{matrix} (a_{j,}\alpha_j)_{1,n}; (a_{ji,}\alpha_{ji})_{n+1,p_i} \\ (b_{j,}\beta_j)_{1,m}; (b_{ji,}\beta_{ji})_{m+1,q_i} \end{matrix} \right.\right]$$
$$= \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1 - b_{ji} + \beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji} - a_{ji})\right\}} \mathrm{Z^s ds}$$

... (3.1)

where $p_I(i = 1, \dots, r), q_I(i = 1, \dots, r), m, n$are integers satisfying $0 \le n \le p_{i,} 0 \le m \le q_i(i = 1,2 \dots . r)$;r is finite,$\alpha_j, \beta_{j,}\alpha_{ji,}\beta_{ji}$are real and positive and $a_j, b, a_{ji,}b_{ji}$ are complex numbers such that $a_j(b_h + v) \ne \beta_h(a_j - 1 - k)$ for v,k=0,1,2,.....,m;,i=1,2,....,r.The contour extend from $+i\infty to - i\infty$such that all the poles of $\Gamma(b_{j,}\beta_j s)$ for j=1,2,...,m and those for $(1 - a_j + \alpha_j s)$ for j=1,....,n are separated by the contour.

5. The Gaussian Hypergeometric Function and its Generalizations

John Wallis, in his work Arithmetical Infinitorum in 1655, first used the term 'hypergeometric' (from the Greek word νπερ, above or beyond) to denote any series which was beyond the ordinary geometric series 1 + x + x 2 + ⋯ In particular, he studied the series 1 + a + a(a + 1) + a(a + 1)(a + 2) + ⋯

Because of the many relations connecting the special functions to each other and to the elementary functions, it is natural to enquire whether more general functions can be developed so that the special functions and elementary functions are merely specializations of these general functions. General functions of this nature have in fact been developed and are collectively referred to as functions of the hypergeometric type. There are several varieties of these functions, but the most common are the hypergeometric functions. Some important results concerning the hypergeometric function had been developed earlier by Euler and others, but it was famous German mathematician C.F. Gauss who in 1812, studied the following infinite series which is generalization of the elementary geometric series and popularly known as Gauss series or more precisely Gauss hypergeometric series [22-25].

∑ (a)n (b)n/ (c)nn! xn ∞ n=0 = 1 + a. b 1. cx + (a + 1). (b + 1) 1.2. (c + 1) x 2 + ⋯
where (a)n = ∏ (a + k − 1) nk =1 = a (a + 1) (a + 2) … (a + n − 1) is the Factorial function, or if a> 0 then (a)n = Γ(a+n) Γ(a) (where Γ is Euler's Gamma function), obviously (a)0 = 1 and (a)n = n! Gauss represented this series by the symbol F1(a, b; c; x) 2 and called it the hypergeometric function. Here x is a real or complex variable, a, b and c are parameters having real or complex

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values and $\neq 0, -1, -2, \ldots .F1(a, b; c; x)\ 2 = \sum (a)n\ (b)n\ (c)nn!\ \infty\ n=0$.

We aim to introduce fractional calculus as a new tool for modeling the complex systems, especially viscoelastic material. First, we briefly discuss the basic concepts of fractional calculus and explain the essential steps of the fractionalization algorithm. Next, we present an interpretation of fractional derivative and elaborate upon how fractional equations could be solved analytically. Then, we briefly look at the modeling of viscoelastic systems by the help of this approach. Ultimately, after overviewing some recent works, we present an application of the approach in modeling biomechanical properties of a cell and indicate that the proposed model predicts the cell behavior much better than the previous spring-dashpot models, as well as the model outputs are in good agreement with experimental data. To sum, we are going to give the minimum need to get reader "feet wet", so that a reader can quickly get into building a fractional calculus model for a complex system [26,27]

Fractional calculus has its origin in the question of extension of meaning. For example, extension of real numbers to complex numbers, factorials of natural numbers to generalized factorials or gamma functions and many such others. The original question that led to the name of fractional calculus was: Can the meaning of derivative of integer order $dnydxn$ be extended to have a meaning when n is a fraction? Later the question became: Can n be any number, fractional, irrational or complex? Because this question was answered affirmatively, the name fractional calculus has become a misnomer and might better be called integration and differentiation to an arbitrary order. Many distinguished mathematicians attended these conferences. These luminaries included R. Askey, M. Mikolas, M.Al-Bassam, P. Heywood, W. Lamb, R. Bagley, Y.A. Brychkov, R. Gorenflo, S.L. Kalla, E.R. Love, K. Nishimoto, S. Owa, A.P. Prudnikov, B. Ross, S. Samko, H.M. Srivastava, J.M.C. Joshi and many others. The papers on the fractional calculus and generalized functions, inequalities obtained by use of the fractional calculus and applications of the fractional calculus to probability theory presented in the conference were quite electric.
$dny/dxn = m!\ (m - n)!\ xm-, (m \geq n,\ n \in N,\ m \in N0)$

Fractional calculus is a field of Mathematical study that branches out of the traditional definition of calculus integral and derivative operators in the same way fractional exponents is an outgrowth of exponents with integer value.

The idea of fractional calculus i.e., fractional derivatives and fractional integrals is not new. In 1695 L'Hospital inquired the question as to the meaning of n dx/dy if 2 1 n =; that is" what if n is fractional?". Leibniz replied that "2 1 2 1 dx/dy will be equal to x dx: x". Usually it is known that integer-order derivatives and integrals have apparent physical and geometric interpretations. However, in case of fractional-order integration and differentiation, which symbolize a rapidly growing field both in theory and in applications to real world problems, it is not so. Since the emergence of the idea of differentiation and integration of arbitrary order, for more than 300 years there was not any adequate geometric and physical interpretation of these operations. In Podlubny (2016), it was made known that geometric interpretation of fractional integration is "gloom on the fortifications" and its Physical explanation is "gloom of the precedent. In the past few years, the use of function has been in studies of visco-elastic materials, as well as in many areas of science and engineering including fluid flow, rheology, diffusive transport, electrical networks, electromagnetic theory and probability. In the present chapter various definitions of fractional derivatives and integrals (differ-integrals) would be considered. Explicit formula of fractional derivative and integral are accessible for several elementary functions. Fractional calculus is applied to dynamical systems in control theory, electrical [28-30].

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6. Conclusion

Developed certain generalized fractional integral operators involving multivariable H-Function, I-Function and general class of polynomials, thereby generalizing several important results obtained earlier in the literature. Some theorems pertaining to N fractional calculus of product involving I-Function and $\bar{H}$-Function have been established, thereby providing unification and extension of several (known and new) results lying hit hereto in the literature. Some new integrals pertaining to I-Function and Wright's Generalized Hypergeometric function have been established which are believed to be useful in mathematical analysis, both pure and applied. Some theorems exhibiting integral properties of Aleph (ℵ)-Function have been established involving product of Aleph (ℵ)-Function and generalized polynomials which find their applications in fluid flow, Rheology, electric networks and probability etc. Presented a mathematical model involving I-Function to study the effect of environmental pollution on the growth and existence of biological populations and applied both Laplace and Sumudu transforms to $\bar{H}$-Function in view of presenting a comparative study.

References:

[1]. Agarwal, R.P., A proposd'une note de M.Pierre Humbert, C.R. Séances Acad. Sci., 236 (21) (1953):2031-2032.

[2]. Agarwal, Suthar, Tadesse, Habenom, 2021, Certain k-Fractional Calculus Operators and Image Formulas of Generalized k-Bessel Function.

[3]. Alshabanat, Jleli, Kumar, Samet, 2021, Generalization of Caputo-Fabrizio Fractional Derivative and Applications to Electrical Circuits.

[4]. Assaleh K. and Ahmad W.M., Modeling speech signals using fractional calculus, 9th international symposium on signal processing and its applications, ISSPA,12 – 15 Feb., (2007): 1-4.

[5]. Baleanu, Agarwal, 2021, Fractional Calculus in the Sky.

[6]. Baleanu, Fernandez, 2019, Fractional Operators and Their Classifications.

[7]. Carpinteri A., Chiaia, B. and Cornetti P., Static-Kinematic duality and the principle of virtual work in the mechanics of fractal media, Computer Methods in Applied Mechanics and Engineering, 191 (2001):3-19

[8]. Chen, Farid, Rehman, & Latif, 2020, Estimations of Fractional Integral Operators for Convex Functions and Related Results.

[9]. Dass, Shantanu, Functional Fractional Calculus for System of Identification and Controls, Springer – Verlag Berlin Heidelberg, 2008.

[10]. Duff, G.F.D and Naylor, D.: Differential Equations of Applied Mathematics, Wiley, (1966):118-122.

[11]. Erdelyi A.(ed.), Higher Transcendental Functions, McGraw-Hill, New York,3(1955).

[12]. Erdelyi A. (ed.), Tables of Integral Transforms, McGraw-Hill, York,1 (1954).

[13]. Fahad, Fernandez, Rehman, & Siddiqi, 2020, Tempered and Hadamard-Type Fractional Calculus with Respect to Functions.

[14]. Gorenflo, R. and Mainardi, F., Fractional Calculus: Integral and Differential Equations of Fractional Order, Springer Verlag Wien and New York, (1997):223-276.

[15]. G. Jumarie, 2016, On the Non-Differentiability of the Solutions of the Fractional Differential Equation $\boldsymbol{y}^{(\boldsymbol{\alpha})}$ $(\boldsymbol{x})$ = $\boldsymbol{\lambda y}(\boldsymbol{x})$.

[16]. G. Jumarie, "On the solution of the stochastic differential equation of exponential growth driven by fractional Brownian motion," Applied Mathematics Letters, vol. 18, no. 7, pp. 817–826, 2005.

[17]. G. Jumarie, "An approach to differential geometry of fractional order via modified Riemann-Liouville derivative," Acta Mathematica Sinica, vol. 28, no. 9, pp. 1741–1768, 2012.

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Corresponding author. E-mail addresses: sachindhewa@gmail.com

[18]. G. Jumarie, “On the derivative chain-rules in fractional calculus via fractional difference and their application to systems modelling,” Central European Journal of Physics, vol. 11, no. 6, pp. 617–633, 2013.
[19]. Gill & Modi, 2018, On Analytic Solutions of Fractional Differential Equation Associated with RLC Electrical Circuit.
[20]. Hartley, T.T. and Lorenzo, C.F., A Solution to the Fundamental Linear Fractional Order Differential Equations, NASA/TP-1998-208963, 1998.
[21]. Humbert, P. and Agarwal R.P., Sur la function de Mittag-Lefferetquelques-unes de ses generalizations, Bulletin des Sciences Mathematiques, 77(10) (1953):180-185.
[22]. H. J. Haubold, A. M. Mathai & R. K. Saxena, 2015, On Mittag-Leffler Functions and Their Applications.
[23]. H. M. Srivastava and H. Exton, On Laplace’s linear differential equation of a general order. Nederl. Akad. Wetensch. Proc. Ser. A 76 = Indag. Math., 35 (1973), 371-374.
[24]. H. M. Srivastava and H. L. Manocha, A Treatise on Generating Functions, Halsted Press (Ellis Horwood Limited, Chichester) 1984.
[25]. H. M. Srivastava and J. P. Singhal, A unified presentation of certain classical polynomials. Math. Comput. 26 (1972) 969-975.
[26]. H. M. Srivastava, K. C. Gupta and S. P. Goyal, The H-function of one and two variables with applications, South Asian publishers, New Delhi and Madras (1982).
[27]. H. M. Srivastava and M. Garg, some integrals involving general class of polynomials and the multivariable H-function. Rev. Roumaine. phys. 32 (1987) 685-692.
[28]. H. M. Srivastava, M. Saigo, and S. Owa, A class of distortion theorems involving certain operators of fractional calculus, J. Math. Anal. Appl., 131(1988), 412-420.
[29]. K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley & Sons, New York, NY, USA, 1993.
[30]. K. B. Oldham and J. Spanier, The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order, Academic Press, New York, NY, USA, 1974.

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