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Bianchi type III bulk viscous string cosmological model in Lyra manifold

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In this paper, we have investigated a Bianchi type III string cosmological model with bulk viscosity in lyra's geometry. to get the realistic solution, we assumed two physically plausible conditions (i) shear scalar (σ) proportional to the expansion factor (θ) which leads to $P = Q^n$; $n \neq 0$ is a constant and P & Q being scale factors and (ii) $\xi = \xi_0 =$ constant, ξ being the coefficient of bulk viscosity. The physical and geometrical properties of the universes are discussed in detail. Interestingly, it is established that the bulk viscosity acts as a crucial role in the evolution of the universe and our model can be treated as a model of realistic universe.

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I. INTRODUCTION

The observations of the universe with the aid of modern technological tools also prove that in the early Universe there exists a large scale network of strings. These strings posses stress-energy and are coupled with the gravitational field. An Anisotropy of the universe is due to the presence of strings; however, strings are not visible nowadays. Strings do not pose a threat to the cosmological models, however, they show the way to incredibly exciting astrophysical consequences, as opposed to domain walls and monopoles. Strings are also useful to explain both the nature and fundamental configuration of the early Universe. In String theory, all the matters and forces are combined to give us a single theoretical structure. The theory explains the initial state of the formation of Universe in terms of (vibrating) strings. The string theory is very appropriate in describing the early evolution of the universe. Many researchers nowadays are focusing on studying cosmological models with string of the universe, to gain a comprehensive understanding of the development of the universe. According to the GUT (grand unified theories) [1, 2, 3, 4, 5, 6, 7], after the big bang explosion, these strings arose when the cosmic temperature goes down below some critical temperatures due to symmetry breaking throughout the phase transition in the early Universe. These cosmic strings can couple toward the gravitational field and posses stress energy. Therefore, it may be fascinating to study the gravitational effect due to strings.

From the recent observations [8, 9, 10] it can be said that the space-time universe is not perfectly isotropic rather it is anisotropic. Therefore, the metric components in the line element should be different functions of time. To study the homogeneous and anisotropic cosmological model with anisotropic property, Bianchi type space-times are used and where isotropization process of these models may also be studied. From the theoretical perception, the anisotropic cosmological models have possessed a better generalization than the isotropic model of the universes. Considering the spatial isotropy, [11, 12] shown that the Bianchi-type string cosmological model generalizes FRW models over and above those with asymptotic or less than SO (3) isotropy.

The paper is structured as follows: Section I is introductory in nature. Section II deals with model and its basic formalism. In section III, we describe the solution of field equations. The physical as well as geometrical properties of the model are discussed in section IV. In section V, we summarized the main results of this paper.

II. THE METRIC AND FIELD EQUATIONS

THe Bianchi type - III space-time for five dimensional universe is read as

$$ds^2 = A^2\left(dx^2 + e^{-2\alpha x}dy^2 + dz^2\right) + B^2 dm^2 - dt^2 \quad (1)$$

where A(t) & B(t) are the scale factors, $\alpha \neq 0$ is a constant, 'm' is space like fifth coordinate [?] and the spatial curvature is taken as zero [13].

Einstein's field equations for Lyra's manifold in normal gauge is read as

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi^k\phi_k = -T_{ij} \quad (2)$$

with $\frac{8\pi G}{c^2} = 1$ in geometrical unit, where R_{ij}, R, T_{ij} and ϕ_i are respectively the Ricci tensor, Ricci scalar, energy momentum tensor and displacement vector field. Let us define ϕ_i as

$$\phi_i = (0, 0, 0, 0, \beta(t)) \quad (3)$$

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Energy-momentum tensor T_{ij} for bulk viscous fluid is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j) \quad (4)$$

where, $\rho = \lambda + \rho_p$ is the energy density for a cloud of strings (loaded with particle), λ denotes the string tension density, ρ_p the particle energy density, $\theta = u^k_{;k}$ is the expansion factor, ξ is the bulk viscosity coefficient, x^i is the unit space like vector representing the direction of strings and u^i is the five velocity vector of fluid flow.

The velocity vector u^i and direction of string x^i are given by

$$u^i = (0, 0, 0, 0, 1) \quad (5)$$

and

$$x^i = (0, 0, 0, \frac{1}{Q}, 0) \quad (6)$$

In co-moving coordinates, the velocity vector u^i and direction of string x^i satisfy the conditions

$$u_i u^i = -x_i x^i = -1 \quad and \quad u^i x_i = 0 \quad (7)$$

For the line element (1), essential physical parameters like the spatial volume V, the average scale factor R, the expansion factor θ, the Hubble expansion factor H, the shear scalar σ^2 and the mean anisotropy parameter Δ are found as follows:

$$V = R^4 = A^3 B \quad (8)$$

$$\theta = u^i_{;i} = 3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \quad (9)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[3\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{1}{4}\theta^2\right] \quad (10)$$

$$4H = \theta = u^i_{;i} = 3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \quad (11)$$

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \quad (12)$$

where $H_j; j = 1, 2, 3, 4$ are respectively the directional Hubble's parameter along the direction of x, y, z & m which are defined as

$$H_1 = H_2 = H_3 = \frac{\dot{P}}{P} \quad and \quad H_4 = \frac{\dot{Q}}{Q}$$

Henceforth, overhead dots are used to denote derivatives with respect to time 't'.

Using equations (1)-(7), in the system of co-moving coordinate, we have the survival field equations as

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = \xi\theta \quad (13)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = \xi\theta \quad (14)$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} + \frac{3}{4}\beta^2 = \lambda + \xi\theta \quad (15)$$

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} - \frac{3}{4}\beta^2 = \rho \quad (16)$$

III. SOLUTION OF THE FIELD EQUATIONS

Considering $\xi = \xi_0(constant)$, the determinate solutions of the above field Eqns. (13)-(16) are obtained in the following section.

Since there are four highly nonlinear independent Eqns. (13)-(14) involving 6 (six) unknown variables (P, Q, β, λ, ρ and θ), therefore to get determinate solutions of the above system of Eqns. we need two extra equations comprising them. To obtain two extra equations, we assume the following physically plausible conditions.

(i) Considering shear scalar (σ) proportional to expansion factor (θ), it can be obtained that

$$A = B^n \quad (17)$$

where $n \neq 0$ is a constant.
The above assumption is based on observations of velocity and red-shift relation for an extragalactic source which predicted that the Hubble expansion is 30% isotropic.

(ii) Also, Berman's [14] suggestion regarding variation of Hubble's parameter H provides us a model of the universe that expands with constant deceleration parameter defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = constant \quad (18)$$

It is well known that whenever q (deceleration parameter) is negative then the model universe is expanded with acceleration, whereas a positive q explains a decelerating universe. Although the present observations like CMBR and SNe Ia suggested the negative value of q (accelerating model universe) but it can be remarkably state that they are not able to deny about the decelerating expansion of universe. Vishwakarma [15] shown that decelerating universe are also not inconsistent by means of these observations.

Solving Eq. (18) for $q = constant$, it can be found that

$$R = (at+b)^{\frac{1}{1+q}} \tag{19}$$

where $a \neq 0$ & b are integrating constants and $q > -1$ for accelerating model universe.

Now from equations (8), (17) and (19) we have

$$A = (at+b)^{\frac{4n}{(3n+1)(1+q)}} \tag{20}$$

$$B = (at+b)^{\frac{4}{(3n+1)(1+q)}} \tag{21}$$

Taking $a = 1$ and $b = 0$ in equations (20) and (21), which do not affect the generality, by the suitable choice of coordinate, so that the equations (20) and (21) reduces to

$$A = t^{\frac{4n}{(3n+1)(1+q)}} \tag{22}$$

$$B = t^{\frac{4}{(3n+1)(1+q)}} \tag{23}$$

IV. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

Energy density ρ is obtained from Eq. (13) & (16) as follows

$$\rho = -\frac{4\xi_0}{(1+q)t} - \frac{4(2n+1)(q-3)}{(3n+1)(1+q)^2t^2} - \alpha^2 t^{\frac{-8n}{(3n+1)(1+q)}} \tag{24}$$

From equations (13) and (15), the value of λ (tension density) is obtained as

$$\lambda = -\frac{4(n-1)(q-3)}{(3n+1)(1+q)^2t^2} - \alpha^2 t^{\frac{-8n}{(3n+1)(1+q)}} \tag{25}$$

Therefore, by using the equations (25) and (26) in the relation $\rho = \lambda + \rho_p$, the energy density of particle is found as follows

$$\rho_p = -\frac{4\xi_0}{(1+q)t} - \frac{8(n+2)(q-3)}{(3n+1)(1+q)^2t^2} \tag{26}$$

The displacement vector β can be obtained from equation (13) as

$$\beta^2 = \frac{16\xi_0}{3(1+q)t} -$$

$$\frac{64(3n^2+2n+1) - 16(2n+1)(3n+1)(1+q)}{3(3n+1)^2(1+q)^2t^2} \tag{27}$$

From Eq. (8) - (12) and Eq. (19) & (24), the physical and geometrical quantities like V , R, θ , H , σ^2 and Δ are obtained as follows:

$$R = t^{\frac{1}{1+q}} \tag{28}$$

$$V = t^{\frac{4}{1+q}} \tag{29}$$

$$\theta = \frac{4}{(1+q)t} \tag{30}$$

$$H = \frac{1}{(1+q)t} \tag{31}$$

$$\sigma^2 = \frac{6(n-1)^2}{(3n+1)^2(1+q)^2t^2} \tag{32}$$

and

$$\Delta = \frac{3(n-1)^2}{(3n+1)^2} \ \ (= constant) \tag{33}$$

From the expressions of average scale factor R, volume V & expansion scalar θ we observe that the model of the universe begin with initial singularity at $t = 0$ from $V = 0$ i.e. our model universe starts from zero volume at $t = 0$ with big-bang & as time progresses it is expanding i.e when $t \to \infty$, $V \to \infty$. Again, from Eq. (31), it is seen that $\frac{dH}{dt}$ is negative, indicating that the expansion rate of model is accelerating but when $t \to \infty$, $\theta \to 0$ showing that the expansion of our stops at infinite time.

From the expressions of energy density ρ and tension density λ given by Eq. (24) and (25), we have observed that both of them are negative at the initial epoch of time but as the time progresses they will change sign from negative to positive and finally becomes zero when $t \to \infty$.

V. CONCLUSION

Investigating a homogeneous and anisotropic space-time described by five - Dimensional Bianchi type-III metric in Lyra geometry with constant deceleration parameter. The exact solutions of Einstein's field equations have been obtained. In this model, the universe starts evolving with zero volume at time t = 0 with Big - Bang and is expanding with acceleration. During the evolution of universe, it has been observed that strings are disappeared leaving only the particles. Also, the shear scalar σ become zero as $t \to \infty$. So, the derived model of the universe represents a shear free inflationary cosmological model that describes the late time phenomenon of the universe.

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